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Abstraction, Re-Presentation, and Reflection: An Interpretation of Experience and of Piaget's Approach*

> The understanding, like the eye, whilst it makes us see and perceive all other things, takes no notice of itself; and it requires art and pains to set it at a distance and make it its own object. (John Locke, 1690)[1]

> As adults we are constantly deceiving ourselves in regard to the nature and genesis of our mental experiences. (John Dewey, 1895)[2]

One of the remarkable features of the Behaviorist era in American psychology is that so many leaders and followers of that creed could claim to be Empiricists, cite John Locke as their forefather, and get away with it. Had they read the first chapter of "Book II"[3] of his major work, An Essay Concerning Human Understanding, they would have found, among many others, the following enlightening statements. Paragraph 2 has the heading: "All Ideas come from Sensation or Reflection." Paragraph 4 has the heading "The operations of our Minds", and it is there that Locke explains what he means by "reflection":

> This source of ideas every man has wholly in himself; and though it be not sense, as having nothing to do with external objects, yet it is very like it, and might properly enough be called internal sense. But as I call the other Sensation, so I call this Reflection, the ideas it affords being such only as the mind gets by reflecting on its own operations within itself.

In our century, it was Jean Piaget who vigorously defended and expanded the notion of reflection. He lost no opportunity to distance himself from empiricists who denied the mind and its operations and wanted to reduce all knowing to a passive reception of objective "sense data". Yet, he should not have found it difficult to agree with Locke's division of ideas because it is not too different from his own division between figurative and operative knowledge. Similarly, I feel, Locke would have had a certain respect for Piaget's effort to set understanding at a distance and to make it the object of investigation. And both men, I have no doubt, would have agreed with Dewey about the risk of deceiving oneself by taking mental experiences as given. It is therefore with caution that I shall proceed to discuss, in the pages that follow, first my own view of reflection, abstraction, re-presentation, and the use of symbols, and then a tentative interpretation of Piaget's position. If, at times, I may sound assertive, I

would beg the reader to keep in mind that I am fully aware of the fact that I am merely offering conjectures—but they are conjectures which I have found useful in constructing a model of mental operations.

Reflection

If someone, having just eaten an apple, takes a bite out of a second one, and is asked which of the two tasted sweeter, we should not be surprised that the person could give an answer. Indeed, we would take it for granted that under these circumstances any normal person could make a relevant judgment. We cannot observe how such a judgment is made. But we can hypothesize some of the steps that seem necessary to make it. The sensations that accompanied the eating of the first apple would have to be remembered, at least until the question is heard.[4] Then they would have to be re-presented and compared (in regard to whatever the person called "sweetness") with the sensations accompanying the later bite from the second apple. This re-presenting and comparing is a way of operating that is different from the processes of sensation that supplied the material for the comparison. Reflecting upon experiences is clearly not the same as having an experience. In 1795, a hundred years after Locke, Wilhelm von Humboldt jotted down a few aphorisms which, posthumously, his editors put under the heading "About Thinking and Speaking". The first three aphorisms deal with reflection:

1. The essence of thinking consists in reflecting, i.e., in distinguishing what thinks from what is being thought.

2. In order to reflect, the mind must stand still for a moment in its progressive activity, must grasp as a unit what was just presented, and thus posit it as object against itself.

3. The mind then compares the units, of which several can be created in that way, and separates and connects them according to its needs. (1907, Vol.7, part 2; p. 581)[5]

I know of no better description of the mysterious capability that allows us to step out of the stream of direct experience, to re-present a chunk of it, and to look at it as though it were direct experience, while remaining aware of the fact that it is not. I call it mysterious, because, although we can all do it as easily as flipping a switch, we have not even the beginnings of a model (least of all an "information processing" model) that would suggest how it might be achieved. "To grasp as a unit what was just presented" is to cut it out of the continuous experiential flow. In the literal sense of the term, this is a kind of abstraction—namely the simplest kind. Focused attention picks a chunk of experience, isolates it from what came before and from what follows, and treats it as a closed entity. For the mind, then, "to posit it as object against itself", is to re-present it. In the next two sections, I want to deal with abstraction and re-presentation one after the other.

Abstraction

As von Humboldt stated in his third aphorism, chunks of experience, once isolated, can be compared, separated, and connected. This makes possible further steps of abstraction, among them the kind that Piaget and many others have called "generalizing abstraction". Because it seems crucial in all forms of naming and

categorization, it has been discussed for a long time. To clarify the core of the notion, I once more return to Locke, because he produced a very simple and widely accepted description of the process:

> This is called Abstraction, whereby ideas taken from particular beings become general representations of all the same kind; and their names general names, applicable to whatever exists conformable to such abstract ideas. (ibid. Book II, Ch. X, §9)

Locke's use of the words "being" and "exist" in this context caused Berkeley, who had a very different view of "existence", to voice a sarcastic objection against his predecessor.

> Whether others have this wonderful faculty of abstracting their ideas, they best can tell; for myself, I find indeed I have a faculty of imagining, or representing to myself, the ideas of those particular things I have perceived, and of variously compounding them. I can imagine a man with two heads, or the upper parts of a man joined to the body of a horse, I can consider the hand, the eye, the nose, each by itself abstracted or separated from the rest of the body. But then whatever hand or eye I imagine, it must have some particular shape and colour. (1710; Introd., p.10)

This passage is interesting for two reasons. Berkeley claims, much as did von Humboldt, that we are able to represent to ourselves particular experiential items and that we are also able to segment them and to recombine the parts at will. Then however, he goes on to claim that whatever we re-present to ourselves must have the character of a particular—from which he concludes that we cannot have general ideas.

Both these claims concern re-presentation and are, I believe, perfectly valid. But what follows from them is that we are unable to re-present general ideas, not that we cannot have them. Berkeley, it seems, unwittingly trapped himself into this position about abstraction. At the beginning of his Treatise, he says among other things:

> Thus, for example, a certain colour, taste, smell, figure and consistence having been observed to go together, are accounted one distinct thing, signified by the name apple; other collections of ideas constitute a stone, a tree, a book, and the like sensible things. (1710; p.1)

Berkeley, of course, was aware of the fact that he would apply the name "apple" not only to one unique "thing", but to countless others that fitted his description in terms of "colour, taste, smell, figure, and consistence", but to him this arose from the association of the word and it seems that he took the ensuing generalization simply for granted. Had he analysed it the way he analysed other conceptual operations, he might have changed his view about abstraction. I hope to make this clear with the help of an example. A child growing up in a region where apples are red would neessarily and quite correctly associate the idea of redness with the name "apple". A distant relative arriving from another part of the country, bringing a basket of yellow apples, would cause a major perturbation for the child, who might want to insist that yellow things should not be called "apples". However, the social pressure of the family's usage of the word will soon force the child to accept the fact that the things people call "apple" come in different colors. The child might then be told that apples can also be

green, which would enable the child to recognize such a particular green thing as an apple the first time it is brought to the house. Berkeley, I would say, was quite right when he maintained that every time we imagine an apple, it has to have a specific color, but he was wrong to claim that we could, therefore, not have a general idea in our heads that allows us to recognize as apples items that differ in some respects, but nevertheless belong to that class. Hence I suggest that, pace Berkeley, we are quite able to abstract general ideas from experience and that we do this by substituting a kind of place-holder or variable for some of the properties in the sensory complex we have abstracted from our experiences of particular things. I see no reason why one should not call the resulting cognitive structure a concept. Such a structure is more specific with regard to some properties and less specific with regard to others, and it is precisely because of this relative indeterminacy that it enables us to recognize items that we have never seen before as exemplars of a familiar kind. In short, in order to recognize several particular experiential items, in spite of differences they may manifest, as belonging to the same kind, we must have a concept that is flexible enough to allow for a certain variability. That is, instead of specific particulars it must contain variables. Yet it is clear that, in order to "imagine" for instance an apple, we have to decide what color it is to be, because we cannot possibly visualize it red or green or yellow at one and the same time. Berkeley, therefore, was right when he observed that whenever we re-present a concept to ourselves, we find that it is a particular thing and not a general idea. What he did not realize was that the abstraction necessary to recognize things of a kind, does not automatically turn into an image that can be re-presented. The situation, however, is somewhat complicated by our ability to use symbols, but before considering this I want to deal with re-presentation.

Re-Presentation

No act of mental re-presentation, which in this context of conceptual analysis means neither less nor more than the re-generation of a prior experience, would be possible if the original generation of the experience had not left some mark to guide its reconstruction. In this requirement, representation is similar to recognition. Both often work hand in hand, e.g., when one recognizes a Volkswagen though one can see only part of its back but is nevertheless able to visualize the whole. The ability to recognize a thing in one's perceptual field, however, does not necessarily bring with it the ability to re-present it spontaneously. We have all had occasion to notice this. Our experiential world contains many things which, although we recognize them when we see them, are not available to us when we want to visualize them. There are, for instance, people whom we would recognize as acquaintances when we meet them, but were we asked to describe them when they are not in our visual field, we would be unable to recall an adequate image of their appearance. The fact that recognition developmentally precedes the ability to re-present an experiential item spontaneously, has been observed in many areas. It is probably best known and documented as the difference between what linguists call "passive" and "active" vocabulary. The difference is conspicuous in second-language learners but it is noticeable also in anyone's first language: a good many words one knows when one hears or reads them are not available when one is speaking or writing. This lag suggests that having

abstracted a concept that may serve to recognize and categorize a perceptual item is not sufficient to re-present the item to oneself in its absence. Piaget has always maintained that all forms of imaging and re-presenting are, in fact, acts of internalized imitation[6] (Piaget, 1945). In this context, the metaphor of "program" may be useful. A program is the fixed itinerary of an activity that can guide and govern the sequence of its re-enactment. But there are two points to be stressed. First, a program may specify the material on which to act, but it does not supply the material; second, a program may specify what acts are to be performed, but it supplies neither the acting agent nor the action. The first of these limitations, I suggest, may account for the fact that to recognize an experiential item requires less effort than to re-present it spontaneously. This would be so, because in re-presentation not only a program of composition is needed, but also the specific sensory components, which must be expressly generated. In recognition, the perceiver merely has to isolate the sensory elements in the sensory manifold. As Berkeley observed, sensory elements are "not creatures of the will" (1710; p.29). Because there are always vastly more sensory elements than the perceiving agent can attend to and use,[7] recognition requires the attentional selecting, grouping, and coordinating of sensory material that fits the composition program of the item to be recognized. In re-presentation, on the other hand, some substitute for the sensory raw material must be generated. (As the example of the Volkswagen indicates, the re-generation of sensory material is much easier when parts of it are supplied by perception, a fact that was well known to the proponents of Gestalt psychology.) A difference analogous to that between acting on actually present perceptual material, as opposed to acting on material that must itself be generated, arises from the second limiting feature I mentioned. With regard to the need for an acting agent, a program is similar to a map. If someone draws a simple map to show you how to get to his house, he essentially indicates a potential path from a place you are presumed to know to the unknown location. The drawing of the path is a graphic representation of the turns that have to be made to accomplish that itinerary, but it does not and could not show what it is to move and what it is to turn right or left. Any user of the map, must supply the motion and the changes of direction with the focus of visual attention while reading the map. Only if one manages to abstract this sequence of motions from the reading activity, can one transform it into physical movement through the mapped region. (Note that this abstracting and transforming is by no means an easy task for those unaccustomed to map reading.) A program, however, differs from a map in that it explicitly provides instructions about actions and implicitly indicates changes of location through the conventional sequence in which the instructions must be read. (But in the program, too, it is the user's focus of attention, while reading or implementing the program, that supplies the progressive motion.) Also, unlike a map, a program may contain embedded "subroutines" for walking and turning (i.e. instructions how to act), but no matter how detailed these subroutines might be, they can contain only instructions to act, not the actions themselves. In other words, irrespective of how minutely a program's instructions have decomposed an activity, they remain static until some agent implements them and adds the dynamics.

In carrying out a program in an experiential situation, just as in following a map through an actual landscape, the sensory material in the agent's perceptual field can supply cues as to the action required at a given point of the procedure. In the re-

presentational mode, however, attention cannot focus on actual perceptual material and pick from it cues about what to do next, because the sensory material itself has to be re-presented. A re-presentation—at least when it is a spontaneous one—is wholly self-generated (which is one reason why it is usually easier to find one's way through a landscape than to draw a reliable map of it.) The increase of difficulty and the concomitant increase of effort involved in the production of conceptual structures when the required sensory material is not available in the present perceptual field, shows itself in all forms of re-presentation and especially in the re-enactment of abstracted programs of action. Any re-presentation, be it of an experiential "thing" or of a program of actions or operations, requires some sensory material for its execution. That basic condition, I believe, is what confirmed Berkeley in his argument against the "existence" of abstracted general ideas, for it is indeed the case that every time we re-present to ourselves such a general idea, it turns into a particular one because its implementation requires the kind of material from which it was abstracted. This last condition could be reformulated by saying that there has to be some isomorphism between the present construct and what it is intended to reconstruct. Clearly, this isomorphism does not concern a "thing-in-itself" but precisely those aspects one wants to or happens to focus on. As Silvio Ceccato remarked thirty years ago, what we visualize of objects in dreams is no more than is required by the context of the dream.[8] More importantly, this selective isomorphism is the basis of graphic and schematic representations. They tend to supply such perceptual material as is required to bring forth in the perceiver the particular ways of operating that the maker of the graphic or schematic is aiming at. In this sense they are didactic, because they can help to focus the naive perceiver's attention on the particular operations that are deemed desirable. Hence, as I have suggested elsewhere (cf. von Glasersfeld, 1987), they can be divided into iconic and symbolic representations, but neither kind should be confused with the mental re-presentations I am discussing here.

The Power of Symbols

Re-presentations can be activated by many things. Any element in the present stream of experience may bring forth the re-presentation of a past situation, state, activity, or other construct. This experiential fact was called associaion by Hume and used by Freud for his analyses of neuroses. The ability to associate is systematically exploited by language. To possess a word is to have associated it with a representation of which one believes that it is similar to the re-presentations the word brings forth in other users of the language.(Only naive linguists claim that these re-presentations are shared, in the sense that they are the same for all users of the given word.) In my terminology, a word is used as a symbol,[9] only when it brings forth in the user an abstracted generalized re-presentation, not merely a response to a particular situation (cf. von Glasersfeld,1974). Several things, therefore, are indispensable for a word to function as a symbol: (1) the phonemes that compose the word in speech, or the graphic marks that constitute it in writing, must be recognized as that particular item of one's vocabulary. This ability to recognize, as I suggested earlier, is preliminary to the ability to re-present and produce the word spontaneously. (2) The word/symbol must be associated with a conceptual structure that was abstracted from experience

and, at least to some extent, generalized. Here, again, the ability to recognize (i.e. to build up the conceptual structure from available perceptual material) precedes the ability to re-present the structure to oneself spontaneously. Once a word has become operative as a symbol and calls forth the associated meaning as re-presentations chunks of experience that have been isolated (abstracted) and to some extent generalized, its power can be further expanded. By this I mean that, as particular users of the word become more proficient, they no longer need to actually produce the associated conceptual structures as a completely implemented re-presentation, but can simply register the occurrence of the word as a kind of "pointer" to be followed if needed at a later moment. I see this as analogous to the capability of recognizing objects on the basis of a partial perceptual construction. In the context of symbolic activities, this capability is both subtle and important. An example may help to clarify what I am trying to say. If, in someone's account of a European journey, you read or hear the name "Paris", you may register it as a pointer to a variety of experiential "referents" with which you hapen to have associated it—e.g., a particular point on the map of Europe, your first glimpse of the Eiffel Tower, the Mona Lisa in the Louvre—but if the account of the journey immediately moves to London, you would be unlikely to implement fully any one of them as an actual re-presentation. At any subsequent moment, however, if the context or the conversation required it, you could return to the mention of "Paris" and develop one of the associated re-presentations. I have chosen to call this function of symbols "pointing" because it seemed best to suggest that words/symbols acquire the power to open or activate pathways to specific re-presentations without, however, obliging the proficient symbol user to produce the re-presentations there and then. This function, incidentally, constitutes one of the central elements of our theory of children's acquisition of the concept of number (Steffe et al., 1983). In this theory, the first manifestation of an abstract number concept is a demonstration that the subject knows, without carrying out a count, that a number word implies or points to the sequential one-to-one coordination of all the terms of the standard number word sequence, from "one" up to the given word, to some countable items. Indeed, we believe that this is the reason why, as adults, we may assert that we know what, say, the numeral (symbol) "381.517" means, in spite of the fact that we are unlikely to be able to re-present to ourselves an associated particular chunk of experience; we know what it means, because it points to a familiar counting procedure or other mathematical method of arriving at the point of the number line which it indicates. In mathematics this form of symbolic implication is so common that it usually goes unnoticed. For instance, when one is told that the side of a pentagon is equal to half the radius of the circumscribed circle multiplied by $\sqrt{(10-2\sqrt{5})}$, one does not have to draw the square roots to understand the statement—provided one knows the operations the symbols "point to". The potential ability is sufficient, one does not have to carry out the indicated operations. Because it is so often taken for granted that mathematical expressions can be understood without carrying out the operations they symbolize, formalist mathematicians are sometimes carried away and declare that the manipulation of symbols constitutes mathematics.[11]

Piaget's Theory of Abstraction

Few, if any, thinkers in this century have used the notion of abstraction as often and insistently as did Piaget. Indeed, in his view "All new knowledge presupposes an abstraction,..." (Piaget, 1974a; p.89). But not all abstraction is the same. Piaget distinguished two main kinds, "empirical" and "reflective", and then subdivided the second. He has frequently explained the primary difference in seemingly simple terms, for example:

> Empirical abstractions concern observables and reflective abstractions concern coordinations. (Piaget, et al., 1977, Vol.II; p.319).
>
> One can thus distinguish two kinds of abstraction according to their exogenous or endogenous sources;... (Piaget, 1974a; p.81).

Anyone who has entered into the spirit of Genetic Epistemology will realize that the simplicity of these statements is deceptive. The expressions "observables" and "exogenous" are liable to be interpreted in a realist sense, as aspects or elements of an external reality. Given Piaget's theory of knowledge, however, this is not how they were intended. In fact, the quoted passages are followed by quite appropriate warnings. After the first, Piaget explains that no characteristic is in itself observable. Even in physics, he says, the measured magnitudes (mass, force, acceleration, etc.) are themselves constructed and are therefore results of inferences deriving from preceding abstractions (Piaget et al., 1977, Vol.II; p.319). In the case of the second quotation, he adds a little later: "there can be no exogenous knowledge except that which is grasped as content, by way of forms which are endogenous in origin." (Piaget, 1974a; p.91). This is not an immediately transparent formulation. As so often in Piaget's writings, one has to look elsewhere in his work for enlightenment.

Form and Content

The distinction between form and content has a history as long as Western philosophy and the terms have been used in many different ways. Piaget's use of the distinction is complicated by the fact that he links it with his use of "observables" (content) and "coordinations" (forms). "The functions of form and content are relative, since every form becomes content for another that comprises it" (Piaget et al., 1977, Vol.II; p.319). This will make sense, only if one recalls that, for Piaget, percepts, observables, and any knowledge of objects, are all the result of a subject's action and not externally caused effects registered by a passive receiver. In his theory, to perceive, to remember, to re-present, and to coordinate are all dynamic, in the sense that they are activities carried out by a subject that operates on internally available material and produces certain results. A term such as "exogenous", therefore, must not be interpreted as referring to a physical outside relative to a physical organism, but rather as referring to something that is external relative to the process in which it becomes involved. Observation and re-presentation have two things in common: (1) they operate on items which, relative to the process at hand, are considered given. The present process takes them as elements and coordinates them as "content" into a new "form" or "structure"; and (2) the resulting new products can be taken as initial "givens" by a future process of structuring, relative to which they then become "content". Thus, once a process is

achieved, its results may be considered "observables" or "exogenous" relative to a subsequent process of coordination or a higher level of analysis. As Piaget saw, this might seem to lead to an infinite regress (Piaget et al., 1977, Vol.II; p.306), but he put forth at least two arguments to counter this notion. One of them emerges from his conception of scientific analysis. Very early in his career, he saw this analysis as a cyclical program in which certain elements abstracted by one branch of science become the "givens" for coordination and abstraction in another. In an early paper (Piaget, 1929) and almost forty years later in his "classification of the sciences" (Piaget, 1967), he formulated this mutual interdependence of the scientific disciplines as a circle: Biology—Psychology—Mathematics—Physics, and looping back to Biology. Hence, from his perspective, there is no linear progression without end, but simply development of method and concepts in one discipline leading to novel conceptualization and coordination in another. The recent impact of the physics of molecules and particles on the conceptual framework of biology would seem a good example.

Scheme Theory

The second reason against an infinite regress of abstractions is grounded in the developmental basis of Genetic Epistemology and is directly relevant here. The child's cognitive career has an unquestionable beginning, a first stage during which the infant assimilates, or tries to assimilate, all experience to such fixed action patterns (reflexes) as it has at the start (Piaget, 1975; p.180). Except for their initial fixedness, these action patterns function like the schemes which the child a little later begins to coordinate on the basis of experience. Schemes are composed of three elements: (1) an initial experiential item or configuration (functionally linked to what the observer would categorize as "trigger" or "stimulus"), (2) an activity the subject has associated with it, and (3) a subsequent experience associated with the activity as its outcome or result. Schemes thus govern the subject's segmentation and differentiation of experience. When a novel item ("novel" in the observer's judgement) is assimilated to the intitiating element of a scheme, it triggers the associated activity. If the activity leads to the expected result, the acting subject in no way differentiates the item from those that functioned like it in the past. But if, for one reason or another, the activity does not lead to the expected result, this generates a perturbation, which could be described as either disappointment or pleasant surprise. In either case, the perturbation may focus the subject's attention on the configuration that triggered the activity this time, and it may then be discriminated from those past experiences where the activity functioned in the expected manner (cf. Piaget, 1974b; p. 264). If the failure of the scheme and the ensuing discrimination of the novel item or situation leads to the tightening of the criteria of assimilation that determine what can and what cannot be taken as a trigger for the particular scheme, this would constitute an accommodation of the initiating conceptual structure.[10] Similarly, if the outcome was a pleasant surprise, this, too, may lead to an accommodation, in the sense that a new scheme will subsequently be triggered by the newly isolated experience. In infancy, during the child's first two years, i.e., in the sensory-motor period, all this is assumed to take place without awareness and conscious reflection. Yet, the fact that threeor four-month-old infants assimilate items (which, to an observer, are not all the same)

as triggers of a particular scheme, is sometimes described as the ability to generalize. Indeed, this is what animal psychologists, working with rats or monkeys, call "stimulus generalization". With children at a later stage, however, when reflection has begun to operate, the discriminating of experiential items that do function in a given scheme from others that do not, constitutes a mechanism that functions as the source of empirical abstractions that are recognized as such by the acting subject. This, obviously, raises the question when and how the acting subject's awareness is involved. One reason why that question seems quite urgent, is that the word "reflection", ever since Locke introduced it into the human sciences, has implied a conscious mind that does the reflecting. A second reason is that in many places where Piaget draws the distinction between the "figurative" and the "operative", this tends to reinforce the notion that the operative (what Locke described as "the ideas the mind gets by reflecting on its own operations") requires consciousness. As a consequence, it would be desirable to unravel when, in Piaget's theory of the cognitive development, the capability of conscious reflection arises. Piaget himself, as I have said elsewhere (von Glasersfeld, 1982), rarely makes explicit whether, in a given passage, he is interpreting what he is gathering from his observations (observer's point of view), or whether he is conjecturing an autonomous view from the observed subject's perspective (cf. Vuyk, 1981, Vol.II). This difference seems crucial in building a model of mental operations and, therefore, to an understanding of his theory of abstraction and, especially, reflective abstraction. I shall return to this question of consciousness after the next section, where I try to lay out the kinds of abstraction Piaget has distinguished.

Four Kinds of Abstraction

The process Locke characterized by saying, "whereby ideas taken from particular beings become general representations of all the same kind," falls under Piaget's term empirical abstraction. To isolate certain sensory properties of an experience and to maintain them as repeatable combinations, i.e., isolating what is needed to recognize further instantiations of, say, apples, undoubtedly constitutes an empirical abstraction. But, as I suggested earlier, to have composed a concept that can serve to recognize (assimilate) items as suitable triggers of a particular scheme, does not automatically bring with it the ability to visualize such items spontaneously as re-presentations. Piaget makes an analogous point—incidentally, one of the few places where he mentions an empiricist connection:

> But it is one thing to extract a character, x, from a set of objects and to classify them together on this basis alone, a process which we shall refer to as 'simple' abstraction and generalisation (and which is invoked by classical empiricism), and quite another to recognise x in an object and to make use of it as an element of a different (non-perceptual) structure, a procedure which we shall refer to as 'constructive' abstraction and generalisation. (Piaget, 1969; p.317)

The capability of spontaneous re-presentation (which is "non-perceptual", too) develops in parallel with the acquisition of language and may lead to an initial, albeit limited form of awareness. Children at the age of three or four years, are not incapable

of producing some pertinent answer when they are asked what a familiar object is like or not like, even when the object is not in sight at the moment. This suggests that they are able not only to call forth an empirically abstracted re-presentation but also to review it quite deliberately. The notion of empirical abstraction covers a wider range of experience for Piaget than is envisioned in the passage I quoted from Locke. What Locke called "particular beings" were for him "ideas" supplied by the five senses. Because, in Piaget's view, visual and tactual perception involve motion, it is not surprising that the internal sensations caused by the agent's own motion (kinesthesis) belong to the "figurative" and are therefore, for him, raw material for empirical abstractions in the form of motor patterns.[11] That such abstracted motor patterns reach the level where they can be re-presented, you can check for yourself. Anyone who has some proficiency in activities such as running down stairs, serving in Tennis, swinging for a drive in Golf, or skiing down a slope, has no difficulty in re-presenting the involved movements without stirring a muscle. An interesting aspect in such "dry reruns" of abstracted motor experiences is that they don't require specific staircases, balls, or slopes. I mention this because it seems to me to be a clear demonstration of deliberate and therefore conscious re-presentation of something that needed no consciousness for its abstraction from actual experience. This difference is important also in Piaget's subdivision of reflective abstractions to which we turn now. From empirical abstractions, whose raw material is sensory-motor experience, Piaget, as I said earlier, distinguished three types of reflective abstraction. Unfortunately, the French labels Piaget chose for them are such that they are inevitably confused by literal translation into English. The first "reflective" type derives from a process Piaget calls reflechissement, a word that is used in optics when something is being reflected, as for instance the sun's rays on the face of the moon. In his theory of cognition, this term is used to indicate that an activity or mental operation (not a static combination of sensory elements) developed on one level is abstracted from that level of operating and applied to a higher one, where Piaget then considers it to be a reflechissement. (Moessinger & Poulin-Dubois, 1981, have translated this as "projection", which captures something of the original sense.) But Piaget stresses that a second characteristic is required:

> Reflective abstraction always involves two inseparable features: a "reflechissement" in the sense of the projection of something borrowed from a preceding level onto a higher one, and a "reflexion" in the sense of a (more or less conscious) cognitive reconstruction or reorganization of what has been transferred. (Piaget, 1975; p.41)

At the beginning of the first of his two volumes on reflective abstraction (Piaget et al. 1977), the two features are again mentioned:

> Reflective abstraction, with its two components of "reflechissement" and "reflexion", can be observed at all stages: from the sensory-motor levels on, the infant is able, in order to solve a new problem, to borrow certain coordinations from already constructed structures and to reorganize them in function of new givens. We do not know, in these cases whether the subject becomes aware of any part of this. (Piaget et al., 1977, Vol.I; p.6).

In the same passage he immediately goes on to describe the second type of reflective abstraction:

> In contrast, at the later stages, when reflection is the work of thought, one must also distinguish thought as a process of construction and thought as a process of retroactive thematization. The latter becomes a reflecting on reflection; and in this case we shall speak of "abstraction reflechie" (reflected abstraction) or pensee reflexive (reflective thought).

Since the present participle of the verb reflechir, from which both the nouns reflechissement and reflexion are formed, is reflechissante, Piaget used "abstraction reflechissante" as a generic term for both types. It is therefore not surprising that in most English translations the distinction was lost when the expression "reflective abstraction" was introduced as the standard term. The situation is further confounded by the fact that Piaget distinguished a third type of reflective abstraction which he called "pseudo-empirical". When children are able to re-present certain things to themselves but are not yet fully on the level of concrete operations,

> it happens that the subjects, by leaning constantly on their perceivable results, can carry out certain constructions which, later on, become purely deductive (e.g. using an abacus or the like for the first numerical operations). In this case we shall speak of "pseudo-empirical abstraction" because, in spite of the fact that these results are read off material objects as though they were empirical abstractions, the perceived properties are actually introduced into these objects by the subject's activities. (Piaget, et al., 1977; Vol.I, p.6).

To recapitulate, Piaget distinguishes four kinds of abstraction. One is called "empirical" because it abstracts sensory-motor properties from experiential situations. The first of the three "reflective" ones, projects and reorganizes on another level a coordination or pattern of the subject's own activities or operations. The next is similar in that it also involves patterns of activities or operations, but it includes the subject's awareness of what has been abstracted and is therefore called "reflected abstraction". The last is called "pseudo-empirical" because, like empirical abstractions, it can take place only if suitable sensory-motor material is available.

The Question of Awareness

One of the two main results of the research carried out by Piaget and his collaborators on the attainment of awareness, he summarized as follows in La prise de conscience:[12]

> ... action by itself constitutes an autonomous knowledge of considerable power, for while it is only "know-how" and not knowledge that is conscious of itself in the sense of conceptualized understanding, it nevertheless constitutes the source of the latter, because the attainment of consciousness nearly always lags quite noticeably behind this initial knowledge which is remarkably efficacious even though it does not know itself. (Piaget, 1974b; p. 275)

The fact that conscious conceptualized knowledge of a given situation developmentally lags behind the knowledge of how to act in the situation, is commonplace on the sensorymotor level. In my view, as I mentioned earlier, this is analogous to the temporal lag of the ability to re-present a given item relative to the ability to recognize it. But the ability spontaneously to re-present to oneself a sensory-motor image of, say, an apple, still falls short of what Piaget in the above passage called "conceptualized understanding". This would involve awareness of the characteristics inherent in the concept of apple or whatever one is re-presenting to oneself, and this kind of awareness constitutes a higher level of mental functioning. This further step requires a good deal more of what Locke called the mind's "art and pains to set (something) at a distance and make it its own object." A familiar motor pattern is once more a good example: we may be well able to re-present to ourselves a tennis stroke or a golf swing, but few, if any, would claim to have a "conceptualized understanding" of the sequence of elementary motor acts that are involved in such an abstraction of a delicately coordinated activity. Yet it is clear that, insofar as such understanding is possible, it can be built up only as a "retroactive thematization", that is, after the whole pattern has been empirically abstracted from the experience of enacting it.

In Piaget's theory, the situation is similar in the first type of reflective abstraction: he maintains that it, too, may or may not involve the subject's awareness.

> Throughout history, thinkers have used thought structures without having grasped them consciously. A classic example: Aristotle used the logic of relations, yet ignored it entirely in the construction of his own logic. (Piaget & Garcia, 1983; p.37)

In other words, one can be quite aware of what one is cognitively operating on, without being aware of the operations one is carrying out. As for the second type, "reflective thought" or "reflected abstraction", it is the only one about which Piaget makes an explicit statement concerning awareness:

> Finally, we call the result of a reflective abstraction "reflected" abstraction, once it has become conscious, and we do this independently of its level. (Piaget et al., 1977, vol.II; p.303. Emphasis added).

When one comes to this statement in Piaget's summary at the end of the second volume on the specific topic of reflective abstraction, it becomes clear that the sequence in which he usually discusses the three types is a little misleading, because it is neither a developmental nor a logical sequence. What he rightly calls "reflective thought" and lists as the second of three types, describes a cognitive phenomenon that is much more sophisticated than reflective abstractions of type one or type three and, moreover, is relevant also as a further development of empirical abstraction. I would suggest that the two meanings of the word "reflection" be assigned in the following way to Piaget's classification of abstractions: it should be interpreted as projection and adjusted organization on another operational level in the case of reflective abstraction type one and pseudo-empirical abstraction; and it should be taken as conscious thought in the case of reflective abstraction type two (also called "reflected"). In his two volumes La prise de conscience (1974b) and Reussir et comprendre (1974c), there is a wealth of observational material from which Piaget and his collaborators infer that

consciousness appears hesitantly in small steps each of which conceptualizes a more or less specific way of operating. Like von Humboldt, Piaget takes the mind's ability to step out of the experiential flow for granted, but he then endeavors to map when and under what conditions the subject's awareness of its own operating sets in; and he tries to establish how action evolves in its relation to the conceptualization which characterizes the attainment of consciousness (Piaget, 1974b; p.275ff). In the subsequent volume, he provides an excellent definition of what it is that awareness contributes:

> To succeed is to comprehend in action a given situation to a degree sufficient to attain the proposed goals: to understand is to master in thought the same situations to the point that one can resolve the problems they pose with regard to the why and the how of the links one has established and used in one's actions. (Piaget, 1974c; p.237)

The cumulative result of the minute investigations contained in these two volumes enabled Piaget to come up with an extremely sophisticated description of the mutual interaction between the construction of successful schemes and the construction of abstracted understandings, an interaction that eventually leads to accommodations and to finding solutions to problems in the re-presentational mode, i.e., without having to have run into them on the level of sensory-motor experience. In this context, one further thing must be added. In the earlier sections, I discussed the fact that re-presentation follows upon recognition and that the "pointing" function of symbols follows as the result of familiarity with the symbols' power to bring forth re-presentations that are based on empirical abstractions. As the examples I gave of abstracted motor patterns should make clear, symbols can be used, simply to point to such patterns, in which case the re-presentation of action can be curtailed, provided the subject has consciously conceptualized the action and knows how to re-present it. I now want to emphasize that this pointing function of symbols makes possible a way of mental operating that requires conscious conceptualization and, as a result, gives more power to the symbols. Once reflective thought can be applied to the kind of abstraction Piaget ascribed to Aristotle (cf. passage quoted above), there will be awareness not only of what is being operated on but also of the operations that are being carried out. Piaget suggested this in an earlier context:

> A form is indissociable from its content in perception but can be manipulated independently of its content in the realm of operations, in which even forms devoid of content can be constructed and manipulated. ...logico-mathematical operations allow the construction of arrangements which are independent of content ... pure forms ... simply based on symbols. (Piaget, 1969; p. 288; my emphasis.)

In my terms this means, symbols can be associated with operations and, once the operations have become quite familiar, the symbols can be used to point to them without the need to produce an actual re-presentation of carrying them out. If this is accepted as a working hypothesis, we have a model for a mathematical activity that was very well characterized by Juan Caramuel[13], twenty-five years before Locke published his Essay:

> When I hear or read a phrase such as "The Saracen army was eight times larger than the Venetian one, yet a quarter of its men fell on the battlefield, a quarter were taken prisoner, and half took to flight", I may admire the noble effort of the Venetians and I can also understand the proportions, without determining a single number. If someone asked me how many Turks there were, how many were killed, how many captured, how many fled, I could not answer unless one of the indeterminate numbers had been determined. ...
>
> Thus the need arose to add to common arithmetic, which deals with the determinate numbers, another to deal with the indeterminate numbers. (Caramuel, 1670/1977; p.37; original emphasis.)

In Europe, Caramuel says, this "other" arithmetic, which deals with abstractions that are "more abstract than the abstract concept of number," became known as algebra. Given the model of abstraction and reflection I have discussed in these pages, it is not difficult to see what this further abstraction resides in. To produce an actual re-presentation of the operative pattern abstracted from the arithmetical operation of, say, division, specific numbers are needed. This is analogous to the need of specific properties when the re-presentation of, say, an apple is to be produced. But there is a difference: The properties required to form an apple re-presentation are sensory properties, whereas the numbers needed to re-present an "operative pattern" in arithmetic are themselves abstractions from mental operations and therefore re-presentable only with the help of some sensory material. Yet, once symbols have been associated with the abstracted operative pattern, these symbols, thanks to their power of functioning as pointers, can be understood, without the actual production of the associated re-presentation—provided the user knows how to produce it when the numerical material is available.

Conclusion

Abstraction, re-presentation, reflection, and conscious conceptualization interact on various levels of mental operating. In the course of these processes, what was produced by one cycle of operations, can be taken as "given content" by the next one, which may then coordinate it to create a new "form", a new structure; and any such structure can be consciously conceptualized and associated with a symbol. The structure that then functions as the symbol's meaning for the particular cognizing subject, may have gone through several cycles of abstraction and reorganization. This is one reason why the conventional view of language is misleading. In my experience, the notion that word/symbols have fixed meanings that are shared by every user of the language, breaks down in any conversation that attempts an interaction on the level of concepts, that is, attempts to go beyond a simple exchange of soothing familiar sounds.

In analyses like those I have tried to lay out in this chapter, one chooses the words that one considers the most adequate to establish the similarities, differences, and relationships one has in mind. But the meanings of whatever words one chooses are one's own, and there is no way of presenting them to a reader for inspection. This, of course, is the very same situation I find myself in, vis-a-vis the writings of Piaget.

There is no way of discovering what he had in mind—not even by reading him in French. All I—or anyone—could do, is to "interpret", to construct and reconstruct until a satisfactory degree of coherence is achieved among the conceptual structures one has built up on the basis of the read text. This situation, I keep reiterating, is no different from the situation we are in, vis-a-vis our non-linguistic experience, i.e. the experience of what we like to call "the world". What matters there, is that the conceptual structures we abstract turn out to be suitable in the pursuit of our goals; and if they do suit our purposes, that they can be brought into some kind of harmony with one another. This is the same, whether the goals are on the level of sensory-motor experience or of reflective thought. From this perspective, the test of anyone's account that purports to interpret direct experience or the writings of another, must be whether or not this account brings forth in the reader a network of conceptualizations and reflective thought that he or she finds coherent and useful.

Philosophical Postscript

It may be time for a professional philosopher to reevaluate the opposition between empiricism and rationalism. The rift has been exaggerated by an often ill-informed tradition in the course of the last hundred years, and the polarization has led to utter mindlessness on the one side and to various kinds of solipsism on the other. Yet, if we return to Locke, from the partially Kantian position of a constructivist such as Piaget, we may be able to reformulate the difference.

> 24. The Original of all our Knowledge. -In time the mind comes to reflect on its own operations about ideas got by sensation, and thereby stores itself with a new set of ideas, which I call ideas of reflection. These are the impressions that are made on our senses by outward objects that are extrinsical to the mind; and its own operations, proceeding from powers intrinsical and proper to itself, which, when reflected on by itself, become also objects of contemplation—are, as I have said, the original of all knowledge. (Locke, 1690; Book II, Ch.1, p.24)[14]

With one modification, this statement fits well into my interpretation of Piaget's analysis of abstractions. The modification concerns, of course, the "outward objects that are extrinsical to the mind". In Piaget's view, exogenous and endogenous do not refer to an inside and an outside of the organism, but are relative to the mental process that is going on at the moment. The "internal" construct that is formed by the coordination of sensory-motor elements on one level, becomes "external" material for the coordination of operations on the next higher level; and the only thing Piaget assumes as a given starting-point for this otherwise closed but spiraling process, is the presence of a few fixed action patterns at the beginning of the infant's cognitive development. Both Locke's and Piaget's model of the cognizing organism acknowledge the senses and the operations of the mind as the two sources of ideas. Locke believed that the sensory source of ideas, the "impressions" generated by "outward objects", provided the mind with some sort of picture of an outside world. Piaget saw perception as the result of the subject's actions and mental operations aimed at providing, not a picture of, but an adaptive fit into the structure of that outer world. The functional primacy of the two sources, consequently, is assigned differently:

Piaget posits the active mind that organizes sensation and perception as primary, whereas Locke, especially later in his work, tends to emphasize the passive reception of impressions by by the senses. The difference, however, takes on an altogether changed character, once we consider that the concept of knowledge is not the same for both thinkers. For Locke it still involved the notion of Truth as correspondence to an independent outside world; for Piaget, in contrast, it has the biologist's meaning of functional fit or viability as the indispensable condition of organic survival. The difference, therefore could be characterized by saying that classical empiricism accepts without question the static notion of being, whereas constructivist rationalism accepts without question the dynamic notion of living.

Footnotes

* I am greatly indebted to Les Steffe and John Richards for their extensive critique of my manuscript on the same topic, circulated in 1982, and I thank Cliff Konold and Charlotte v. G. for comments on a recent draft of parts of this paper.

1. Locke (1690), Introduction, p. 1.
2. McLellan & Dewey (1895), p.27.
3. Locke divided this work into Chapters, Books, and numbered paragraphs.
4. Memory, as Heinz von Foerster (1965) pointed out, cannot be a fixed record (because the capacity of heads, even on the molecular level, is simply not large enough); hence, it must be thought of as dynamic, i.e., as a mechanism that reconstructs rather than stores.
5. A first English translation of von Humboldt's aphorisms was published by Rotenstreich (1974). The slightly different translations given here are mine.
6. It is crucial to keep in mind that Piaget emphatically stated that knowledge could not be a copy or picture of an external reality; hence, for him, "imitation" did not mean producing a replica of an object outside the subject's experiential field, but rather the re-generation of an externalized experience.
7. Cf. William James (1892/1962; p.227): "One of the most extraordinary facts of our life is that, although we are besieges at every moment by impressions from our whole sensory surface, we notice so very small a part of them."
8. Ceccato said this several times in our discussions on the operations that constitute "meaning" (1947–1952, when I was the translator for his journal Methodos).
9. Note that my use of the word "symbol" is not the same as Piaget's for whom symbols had to have an iconic relation to their referents.
10. Needless to say, the perturbation may simply lead to a retrial or to a modification of the activity; the latter, if successful, would also constitute an accommodation.
11. Having introduced an idea from Ceccato's "operational analyses into Piaget's model, I today believe that the motion necessary in perception need not be physical, but can often be replaced by the motion of the perceiver's focus of attention (cf. von Glasersfeld, 1981).
12. The title of this volume, as Leslie Smith (1981), one of the few conscientious interpreters of Piaget, pointed out, was mistranslated as "The grasp of consciousness" and should have been rendered as "the onset" or "attainment of consciousness".

13. I owe knowledge of Caramuel's work to my late friend Paolo Terzi, who immediately recognized the value of the 17th century Latin treatise, when it was accidentally found in the library of Vigevano. Caramuel, a Spanish nobleman, architect, mathematician, and philosopher of science, had been "exiled" as bishop to that small Lombard city, because he had had several disagreements with the Vatican.
14. It may be helpful to remember that the first sentence in Kant's Critique of pure reason (1781) reads: "Experience is undoubtedly the first product that our intelligence brings forth, by operating on the material of sensory impressions."

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